Application of Integrals

# Work

You may have learned in Physics that if you move an object a distance (*d*) with a force (*F*) it requires work (*W*) to be done. Work is measured in Newton Metres (Nm) or Joules (*J*) and is equivalent to the amount of energy needed to move the object with the required force.

This equation is based on the assumption that the force is constant over the distance. However, this is often not the case in real scenarios. If the force applied at any point *x* is given by the function *F(x)* then the work done in moving it from point *a* to *b* is given by the integral:



Consider the following graphs showing the two cases; constant force and variable force.

A constant force of 5N is applied to move an object a distance of 4. This is the area under the curve from 2 to 6, which is a rectangle of dimensions F x d.

A variable force F(x) N is applied to move an object a distance of 4. This is the area under the curve from 2 to 6, which we need to integrate.

1. **Prove that for a constant force using the general equation , and setting *F*(*x*) to the constant *F*.**
2. **Determine the work done for the variable force applied in the example above.**
3. **Determine the work done when a force of N is used to move an object from to .**

## Hooke’s Law

An example of a variable force is that needed to stretch a spring. The further you try to stretch or compress a spring from it natural length the more force you require. This relationship is expressed by Hooke’s Law: , where *x* is the distance from the natural length and *k* is the spring constant. This Law is usable as long as the spring is not stretched beyond its elastic limit. The spring constant varies for each spring and needs to be calculated if it is not given.

### Example

A spring has a natural length of 10cm and a spring constant of 200. What is the work done to stretch the spring from:

#### Natural length (10cm) to 20cm

­­By Hooke’s Law

Convert measurements to metres: 0.1m to 0.2m

#### 20cm to 30cm

By Hooke’s Law

Convert measurements to metres: 0.2m to 0.3m

Note the greater amount of work done in stretching the spring despite the distance being the same.

1. **A spring of natural length 10cm requires 2J of work to stretch it to twice its length. Determine the spring constant.**
2. **A spring with natural length 5cm and spring constant 100 is stretched using 4Nm of work. To what distance will this stretch the spring?**

## More Work Applications

Consider the example of rolling out a hose reel that is lying on concrete. The further away you drag the hose the more hose you have to move and the greater the weight of hose that you’re dragging. Therefore the force required to move will increase the further you go and so will the work done in moving a given distance. In this situation the coefficient of friction will have a big influence on the force. You can find out more about friction here: <http://en.wikipedia.org/wiki/Friction>. Now, let’s say that the hose has a weight of 0.5kg/m and the coefficient of friction with the concrete is 0.4. Take gravity as 9.8ms-2. So, the force due to weight is 4.9N/m and the force due to friction is N/m. The length of hose that we have at any time is *x* m.

So, N. We can now integrate over whatever distance we are interested in.

1. **Calculate the work done in extending the hose from 2m to 8m.**
2. **Now calculate the work done rolling a 20m hose back up. Assume the whole hose has been rolled out in a straight line and the same parameters exist from before.**